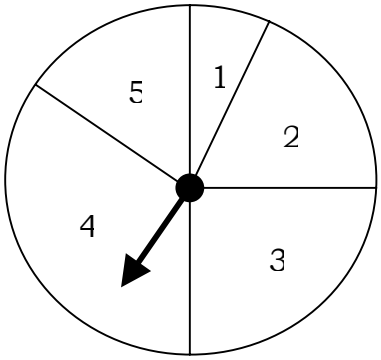
Calculating Theoretical Probabilities

In the previous activity, you determined the likelihood of events associated with tossing a 6-sided number cube, where all faces on the cube were equally likely. In this activity, you will calculate the probability of events that are not equally likely.

Consider the spinner below. Notice that the spinner is divided into five parts and that the parts have different areas.



The following table gives the probability distribution for the spinner.

Outcome	1	2	3	4	5
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$

1. The spinner must land on one of the 5 outcomes, so the probabilities in the table should add up to 1 or 100%. Use mathematics to justify that the sum of the probabilities is 1.

2. Determine the following probabilities using the values in the table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of landing on an even number?

   b. What is the probability of landing on an odd number?

   c. What is the probability of landing on an even or an odd number?

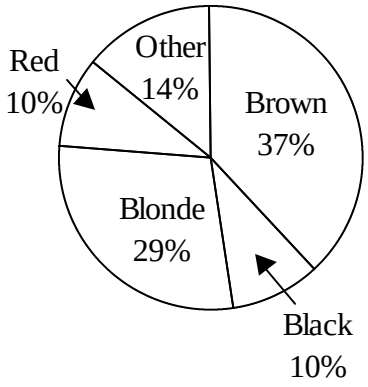
3. Determine the following probabilities using the values in the table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of landing on 2 or 3?

   b. What is the probability of landing on 6?

The pie chart (or circle graph) below gives the distribution of hair colors for twenty-one students in a particular math class.

Hair Color



4. Suppose a student in this math class is chosen at random. Use the information in the pie chart to determine the following probabilities.

   a. What is the probability that the student has blonde hair?

   b. What is the probability that the student chosen has red or black hair?

   c. What is the probability that the student has curly hair?

   d. What is the probability that the student's hair color is *not* brown?

5. Suppose that the students in the math class represent a simple random sample of all students in the school.

   a. What is the probability that a randomly selected student in the school has blonde hair?

   b. What is the probability that a randomly selected student in the school has hair color other than blonde, brown, black, or red?

Many board games involve tossing 6-sided number cubes. Suppose you are playing a game where two number cubes are tossed and the values on the faces of the cubes are added together. The possible outcomes in this game would consist of the possible sums of the two faces of the cubes.

It will be helpful in this activity to list all possible combinations of the two cubes, so that the outcomes are equally likely. Look at the table below.

Faces	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are thirty-six possible combinations of the two cubes. Suppose we wanted to calculate the probability of getting a sum of 4. Three of the thirty-six combinations yield a sum of 4, so $P(4) = 3/36$.

6. Complete the following table by calculating the probability of each possible sum.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability			$\frac{3}{36}$								

7. Determine the probability of each event using the above table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of getting a sum greater than 9?

   b. What is the probability of getting a sum less than or equal to 5?

   c. What is the probability of getting a sum that is an even number?

   d. What is the probability of getting a sum equal to 5, 6, or 7?

   e. What is the probability of getting a sum of 1?

8. Suppose that you observed a sum of 8 on the last three tosses. What is the probability that your next toss will yield a sum of 8? Use mathematics to justify your answer.

Calculating Theoretical Probabilities
Answer Key

1. $\frac{1}{12}+\frac{1}{6}+\frac{1}{4}+\frac{1}{3}+\frac{1}{6}=\frac{1}{12}+\frac{2}{12}+\frac{3}{12}+\frac{4}{12}+\frac{2}{12}=\frac{12}{12}=1$

2. a. $\frac{1}{6}+\frac{1}{3}=\frac{1}{6}+\frac{2}{6}=\frac{1}{2}=0.50$

   b. $\frac{1}{12}+\frac{1}{4}+\frac{1}{6}=\frac{1}{12}+\frac{3}{12}+\frac{2}{12}=\frac{6}{12}=\frac{1}{2}=0.50$

   c. $0.50+0.50=1$

3. a. $\frac{1}{6}+\frac{1}{4}=\frac{2}{12}+\frac{3}{12}=\frac{5}{12}=0.4167$

   b. 0 The spinner will never land on 6 because 6 is not a value for one of the parts of the spinner.

4. a. 0.29

   b. $0.10+0.10=0.20$

   c. There is not enough information to determine this probability.

   d. $0.14+0.10+0.29+0.10=0.63$

5. a. 0.29

   b. 0.14

6.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

7. a. $\frac{3+2+1}{36}=\frac{6}{36}=\frac{1}{6}=0.667$

   b. $\frac{1+2+3+4}{36}=\frac{10}{36}=\frac{5}{18}=0.278$

   c. $\frac{1+3+5+5+3+1}{36}=\frac{18}{36}=\frac{1}{2}=0.50$

   d. $\frac{4+5+6}{36}=\frac{15}{36}=\frac{5}{12}=0.4167$

   e. 0

8. $\frac{5}{36}=0.138$, the outcomes of the prior tosses do not affect the outcome of the ninth toss. We call this "independence."